**UNIVERSITY OF ELECTRONIC SCIENCE AND TECHNOLOGY OF CHINA**

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**COURSE:** DIGITAL SIGNAL PROCESSING

**PURPOSE:** FINAL REPORT

**TOPIC:** DIGITAL FILTER DESIGN

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**DIGITAL FILTER DESIGN**

**1.DIGITAL FILTER SPECIFICATIONS**

Before discusing about how to design a digital filter, we have to know the concept of filter specification because, it describe how the filter will be and give the requred friquency response of filter.The word specification actually refer to the frequency response specification,the following diagrams, fig 1 describe the specifications of low pass filters.

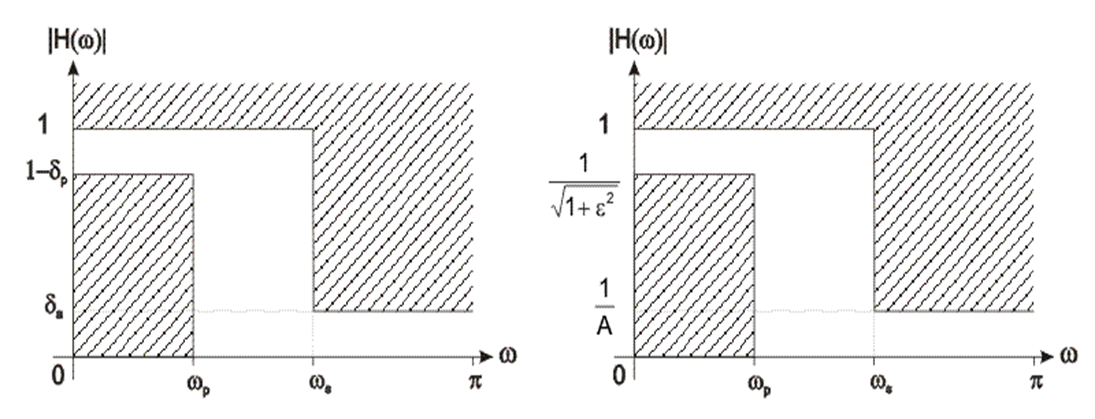


Figure 1: low pass filter

* In the passband, defined by 0 ≤ ω ≤ , we require that with deviation

1-δp ≤ |Ha(jω)| ≤ 1 , |ω| ≤

* In the stopband, defined by ≤ ω ≤ , we require that with deviation

|Ha(jω)| ≤ δs  ≤ ω ≤

**1.1 FILTER SPECIFICATION PARAMETERS**

* -is a passband edge frequency
* - stopband edge frequency
* -peak ripple value in the passband
* -peak ripple value in the stopband

Practical specification are often given in term of loose function (in dB)

Peak passband ripple is given as

* dB

Minimum stopband attenuations

* ) dB

In practice, Passband edge frequency and stopband edge frequency are specified in Hz

For digital filter design, Normalized band edge frequencies need to be computed from specifications in Hz using

and

* - Maximum passband deviation
* - Maximum stopband magnitude
* Transition ratio

Than maximum passband attenuation

* dB

For ,it can be shown that

* dB

**2. IIR DIGITAL FILTER AND FIR DIGITAL FILTER DESIGN**

Filters can be classified in several different groups, depending on what criteria are used for classification. The two major types of digital filters are finite impulse response digital filters (FIR filters) and infinite impulse response digital filters (IIR).In this discussion we will see how to design a digital filter using specifications described above in of those two types of digital filters.

**2.1 The objective of digital filter design is to develop a causal transfer function H (z) meeting the frequency specifications.**

The transfer function of two namely filters are as follow:

1. FIR Digital Filter
2. IIR Digital Filter

**The table below shows the differences between two types of filters.**

|  |  |  |
| --- | --- | --- |
|  | FIR | IIR |
| Impulse Response | finite | Infinite |
| System Function | H(z)=P(z) | H(z)=P(z)/D(z) |
| Structure diagram | Have no feedback | Have feedback |
| Phase response | Exact linear phase  h[n] = ±h[n-N] | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Zero-poles | Only have zeros | Both zeros and poles |

Table:1

Also, the order NFIR of an FIR filter is higher than the order NIIR of an equivalent IIR filter meeting the same magnitude specifications.

The ratio NFIR/ NIIR is typically of the order of 10 or more (the IIR filter usually is computationally more efficient)

**2.1.1 Basic Approaches to Digital Filter Design**

Step1: convert the digital filter specifications into analog lowpass prototype filter specifications

Step2: determine the analog lowpass filter transfer function Ha(s)

Step3: transform Ha(s) into the desired digital filter transfer function G (z)

**Why analog?**

* Analog approximation techniques are highly advanced
* They usually yield closed-form solutions
* Extensive tables are available for analog filter design
* Many applications require the digital simulation of analog filters

**2.2. How to convert an analog prototype transfer function Ha(s) into a digital IIR transfer function G (z)?**

* The imaginary (jὩ) axis in the s-plane be mapped onto the unit circle of the z-plane
* A stable analog transfer function be transformed into a stable digital transfer function

**2.2.1. Estimation of the Filter Order**

IIR: The order of G (z) is determined from the transformation being used to convert Ha(s) into G (z)



**2.2.2. Bilinear Transformation Method of IIR Filter Design**

* Bilinear transformation is more commonly used to design IIR digital filters based on the conversion of analog prototype filters
* The Bilinear Transformation, s-plane to z-plane

**G (z) = Ha(s) |**

* The transformation is a one-to-one mapping. It maps a single point in the s-plane to a unique point in the z-plane

**2.2.3. DIGITAL FILTER DESIGN PROCEDURE:**

* Step1 the invert bilinear transformation is applied to the digital filter specifications to arrive at the specifications of the analog filter function
* Step2 the bilinear transformation is employed to obtain the desired digital transfer function G(z) from the analog transfer function Ha(s) desired to meet the analog filter specifications

**But, When T=2 (T has no effect on the G (z))**

When then 

When then

 Also 

If  then  <1, also If  > 0 then  >1

When and 

, then 

**2.2.4. DESIGN OF LOWPASS IIR DIGITAL FILTERS**

* Step1: get the digital filter specifications (     )
* Step2: convert to analog filter specifications with bilinear transformation
* Step3: design analog transfer function Ha(s)
* Step4: transfer Ha(s) to H (z) since 

**Example;**

Passband edge frequency  is 0.25 with a passband ripple  of 0.5dB

Stopband edge frequency  is 0.55  with a stopband attenuation  of 15dB

Then

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From the passband ripple of 0.5dB obtaining 

From the stopband attenuation of 15dB obtaining 

**Then**



Since

 Then we get 

The transfer function of third-order lowpass Butterworth is

Then we can get



**2.3. DESIGN OF HIGHPASS, BANDPASS, AND BANDSTOP IIR DIGITAL FILTERS**

**2.3.1. To design three types of IIR filters there are two approaches can be followed**

* **First approach:**
* Step1: prewarp the digital frequency specifications to arrive at the specifications of an analog filter of the same type.

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* Step2: convert the frequency specifications of HD(s) into that of a prototype analog lowpass filter HLP(S)



* Step3: Design the analog lowpass filter HLP(S) using the method described
* Step4: convert the transfer function HLP(S) into HD(S) using the inverse of the frequency transformation used in step2
* Step5: Transform the transfer function HD(S) using the bilinear transformation to arrive at the desired digital IIR transfer function GD (Z)



* **The second approach:**
* Step1: prewarp the digital frequency specifications to arrive at the specifications of an analog filter of the same type.

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* Step2: convert the frequency specifications of HD(s) into that of a prototype analog lowpass filter HLP(S)



* Step3: Design the analog lowpass filter HLP(S) using the method described.
* Step4: convert the transfer function HLP(S) into the transfer function GLP(Z) of an IIR digital filter using the bilinear transformation



* Step5: transform GLP(Z) into the desired digital transfer function GD(z) using the appropriate spectral transformation discussed.

**2.3.2. The table below shows the spectral transformations of a low pass filter with a cutoff frequency**

|  |  |  |
| --- | --- | --- |
| FILTER TYPE | SPECTRAL TRANSFORMATION | DESIGN PARAMETERS |
| Lowpass |  | =desired cutoff frequency |
| Highpass |  | =desired cutoff frequency |
| Bandpass |  | =desired upper and lower cutoff frequencies. |
| Bandstope |  | =desired upper and lower cutoff frequencies. |

**Table:2**

**2.4. FIR DIGITAL FILTER DESIGN.**

Unlike IIR digital filter design, FIR filter design does not have any connection with the design of analog filter. The design of FIR filters is there for based on a direct approximation of the specified magnitude response, with the often added requirement that the phase response be linear. A causal FIR transfer function H (z) of length N+I is a polynomial in of degree N.

**2.4.1. FIR Digital filter Order Estimation**

For the design of low pass FIR Digital filter, we can estimating the minimum value of N directly from the digital filter specifications.

There are Three commonly approach used to design FIR filter:-

* Windowed furious series approach
* Frequency sampling approach
* Computer based optimization method

**2.4.2. FIR designs based on window functions.**

FIR filters can also be designed from a frequency response specification. The equivalent sampled impulse response, which determines the coefficients of the FIR filter.

Consider an ideal low-pass characteristic (brick-wall filter) with a cutoff frequency

Where T is a sampling period:

We know that the continuous impulse response is given by (and shown in Fig. 2):

The sampled impulse responseis the sampled version of the continuous function. It is not possible to implement the corresponding low-pass filter design because:

* an infinite number of coefficients would be required
* the impulse response is that of a non-causal system ( exists between and )



Figure 2: Impulse response of brick wall filter in (left) continuous and (right) discrete time domains.

**A first solution**

Hence we could,

1. Truncate the expression for at some reasonable value of say 10.
2. Shift all the coefficients by the same number.

This is shown in Fig 3.

Now that we have the difference equation

For the filter, we can also obtain its transfer function

As before, we can obtain the actual frequency response of the filter by evaluating on the unit circle (i.e. . This is shown in Fig 4 using both Linear and logarithmic plots for the amplitude response.



Figure 3: Impulse response of brick wall filter shifted.



Figure 4: Linear (left) and log (right) responses for 21 and 11 coefficients in the brick wall filter.

**Better solution**

The truncation of the impulse response is equivalent to multiplying it by a rectangular

“Window” function. This leads to an overshoot and ripple before and after the discontinuity in the frequency response – a phenomenon known as Gibb’s phenomenon (the overshoot is about 9% – see previous Fig). The amplitude of the overshoot does not decrease if more and more coefficients are included in the digital filter.

A more successful way of designing an FIR filter is to use a finite weighting sequence. There are a number of such sequences, for example the Hamming, Hanning or Kaiser windows.

If this is the Hamming window, if this is the Hanning, or raised

cosine, window. Fig 5 shows the 11 point Hamming window.



Figure 5: 11 point Hamming window.

The Fourier transform of these windows consists of a central lobe which contains most of the energy and side lobes which generally decay very rapidly. The use of such a window to reduce the Fourier coefficients for the higher frequency terms leads to a reduction in ripple amplitude, at the expense of a slightly worse initial cut-off slope. The frequency response of the 21-coefficient FIR filter Fig. 4 is shown in Fig. 6 together with that of the equivalent “windowed” filter (the filter weights on this case being computed from ) using a Hamming window.

**2.4.3. FIR filter design – conclusion**

FIR filters are usually found in applications where waveform distortion due to non-linear phase is harmful. As the 2 examples of filters studied illustrate, FIR filters with exactly linear phase can be designed (they must have an impulse response which is either symmetrical – i.e. palindromic coefficients – or purely anti-symmetrical). FIR filters are mostly realised as non-recursive structures; such filters are always stable. However, if a sharp cut-off in the amplitude response is required, a large number of coefficients are needed (usually .100).

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Figure 6: Hamming window removing overshoot.

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